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A time-dependent brane in a cosmological background

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Abstract: We study a moving D-brane in a time-dependent background. There is particle production both because of non-trivial cosmological evolution, and by closed string emission from the brane that gradually decelerates due to a gain in mass. The particular model under study is a $D0$-brane in an $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ cosmology — the techniques used extend to other backgrounds.

Keywords: Conformal Field Models in String Theory, Models of Quantum Gravity, Cosmology of Theories beyond the SM.

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Contents

1. Introduction

The study of time-dependent phenomena in string theory has gained momentum, partly under the influence of impressive improvements in the measurement of cosmological data (e.g. the detailed mapping of the cosmic microwave background). The euclidean framework that underlies perturbative string theory does not lend itself easily to interpretation in time-dependent settings. It would seem that any progress we can make in continuing hard euclidean results in string theory into the lorentzian domain, and in interpreting them sensibly, is worthwhile reporting. (See e.g. [1]–[6].)

In this letter, we choose to study an exact cosmological solution to string theory (see e.g. [7]–[9]), exhibiting non-trivial time-dependence in the closed string sector, and we supplement it with a non-trivial moving brane that solves the equations of motion of classical open string field theory. Exact results in the purely closed string background can be obtained from the study of a particular coset conformal field theory, $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$, while exact results in the open string sector can be obtained from analytically continuing boundary states of the euclidean model (see [10]–[12]). Two of the challenges we face is to identify the correct analytic continuation, as well as to interpret correctly the time-dependent physics.

In section 2 we discuss the exact cosmological background, and in part 3 the particle production due to the time-evolution. In section 4 we set up the formalism that will allow us to add the effects due to emission of on-shell closed strings from an extra source. Then, in section 5, we discuss the particular D-brane source we add, and the closed string emission it generates. We wind up with conclusions in section 6.

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2. Cosmological background

We study the cosmology with metric and dilaton

$$\begin{aligned} ds^2 &= |k\alpha'|\frac{dudv}{1-uv} \\ e^{2\Phi} &= \frac{e^{2\Phi_0}}{1-uv} \end{aligned} \tag{2.1}$$

which is known to correspond to an exact two-dimensional conformal field theory, the gauged Wess-Zumino-Witten model $\mathrm{SL}(2,\mathbb{R})_k/\mathrm{U}(1)$, where we consider negative level k. As such, it can be combined with flat space factors, and the euclidean two-dimensional black hole coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ to create a critical (super)string theory background [7].

In the following, we will make good use of a set of alternative coordinate systems, which can be defined as follows (we will neglect the overall factor in the metric on occasion):

$$\begin{aligned} u &= X^1 - X^0 \quad ; \quad v = X^1 + X^0 \\ ds^2 &= \frac{-(dX^0)^2 + (dX^1)^2}{1-(X^1)^2+(X^0)^2} \end{aligned} \tag{2.2}$$

which is useful to study global properties of the spacetime and exhibits the conformally flat nature of the cosmology (up to a non-trivial dilaton factor);

$$\begin{aligned} u &= e^x \sinh t \quad ; \quad v = -e^{-x}\sinh t \\ ds^2 &= -dt^2 + \tanh^2 t\, dx^2 \\ e^{2\Phi} &= \frac{e^{2\Phi_0}}{\cosh^2 t}\,, \end{aligned} \tag{2.3}$$

which is a useful coordinate system for comparison to the euclidean 2-dimensional black hole and

$$\begin{aligned} u &= e^x \tilde{t} \\ v &= -e^{-x}\tilde{t} \\ ds^2 &= \frac{1}{1+\tilde{t}^2}(-d\tilde{t}^2 + \tilde{t}^2 dx^2) \\ |\tilde{t}| &= e^\eta \\ ds^2 &= \frac{e^{2\eta}}{1+e^{2\eta}}(-d\eta^2 + dx^2)\,, \end{aligned} \tag{2.4}$$

which makes manifest the relation of the cosmology to flat space in Milne-like coordinates near the origin $\tilde{t} = 0$.

The spacetime Penrose diagram is drawn in figure 1. There are four regions separated by horizons at $u = 0$ and $v = 0$. Regions I and II are free of any singularities; they can be thought of as expanding and collapsing cosmologies respectively. Region I is defined by $uv < 0$, $X^0 > 0$, while region II by $uv < 0$, $X^0 < 0$. The coordinates defined by equation (2.3) (and also (2.4)) are suitable to parameterize these regions. In terms of the (x,t) coordinates, the horizon corresponds to the surface $t = 0$, and for the cosmological

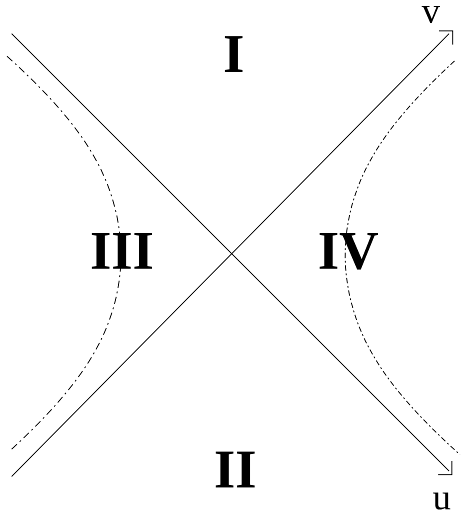

Figure 1: The Penrose diagram for the two-dimensional cosmology.

region I $0 \leq t < \infty$. In regions III and IV ($uv > 0$) (naively) there are timelike naked singularities. No observer in region I can ever meet the singularities; they are simply outside the future lightcone of such an observer. However, it is clear that the singularities can influence the cosmological evolution in region I. In this sense region I is similar to a big bang cosmology. The crucial difference is that near $t = 0$ the spacetime is smooth having no curvature singularity and initial data can be defined on this surface. In the (x, t) coordinates, the metric near $t = 0$ is given approximately by the flat metric in Milne-like coordinates and the string coupling is constant, while as $t \to \infty$ the metric approaches the flat Minkowski metric and the string coupling constant vanishes exponentially. Thus, asymptotically the cosmological region I becomes identical to a timelike linear dilaton background.

We will also make use of the boost symmetry of the cosmology: $(u, v) \to (e^{-x_0}u, e^{x_0}v)$. It is easy to see that in the (x, t) coordinates this symmetry corresponds to translations in the spatial x-direction. Because of the symmetry, the x-momentum is conserved, but due to the time-dependence of the metric there is no conservation law for the energy.

In the following, our idea will be to concentrate on the future region of the cosmology, namely region I. This region naturally Wick rotates to the euclidean black hole (after changing also the sign of the level), which is a well-studied euclidean conformal field theory. We thus concentrate only on this part of the cosmology.[1] It should be noted that different attitudes can be taken and have been taken toward the proper definition of the conformal field theory corresponding to this and other lorentzian backgrounds. One consists in defining the coset strictly in terms of the (hyperbolic) quotient of the original group manifold (see e.g. [14, 9]), and a second consists in following closely techniques of quantum field

[1]The other regions, namely (parts of the) regions III and IV, analytically continue to compact parafermions, SU(2)/U(1). It would be interesting to investigate how to interpret these regions and their parafermionic analytic continuation (and in particular the resulting boundary conditions, spectrum, etc). We thank Costas Kounnas for instructive discussions on these issues.

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theory analysis in curved space for a particular region of the lorentzian space under study. We follow the second method since the euclidean conformal field theory that results from this analysis is under sufficient control to allow for exact results. It is also desirable to study further the direct lorentzian WZW model (and its quotient), and to further compare results obtained through these two complementary methods.

3. Cosmological particle production

In this section, we wish to study particle creation in the bulk of the cosmology for a scalar closed string field ϕ with quadratic effective lagrangian

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}e^{-2\Phi}(-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2)\,. \tag{3.1}$$

In the region of interest, namely the cosmological region I, the dilaton field can be chosen to be arbitrarily small and so as a first step we ignore terms in the effective action describing string interactions. The analysis in this section is equivalent to a mini-superspace analysis of strings propagating in the cosmology, and the results obtained here should match the exact string theoretic results in the limit of infinite negative level. As we shall argue later, some exact string theory results at finite level can be obtained by suitably analytically continuing the corresponding euclidean black hole CFT results. The resulting equation of motion is then given by

$$\left(-\frac{1}{\sqrt{-g}e^{-2\Phi}}\partial_\mu(e^{-2\Phi}\sqrt{-g}g^{\mu\nu}\partial_\nu) + m^2\right)\phi = 0\,. \tag{3.2}$$

As usual the current density (corresponding to a complexified field ϕ)

$$j_\mu = e^{-2\Phi}\sqrt{-g}(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi) \tag{3.3}$$

is conserved and can be used to define an invariant (modified Klein-Gordon) inner product

$$(\phi_1, \phi_2) = -i\int_\Sigma \sqrt{-g}e^{-2\Phi}(\phi_1\partial_\mu\phi_2^* - \partial_\mu\phi_1\phi_2^*)d\Sigma^\mu \tag{3.4}$$

(in notation borrowed from [13]). Due to conservation of the current, the inner product is independent of the choice of the spacelike slice Σ we use to compute it. The modes of the scalar field can thus be normalized using this inner product, and the resulting norm is independent of time.

We now turn to solving the equation of motion. In terms of the (x, t) coordinates, the wave-operator is given by

$$\Box = \frac{1}{|k|\alpha'}(\partial_t^2 + (\coth t + \tanh t)\partial_t - \coth^2 t\partial_x^2)\,, \tag{3.5}$$

where we reintroduced the level k of the conformal field theory. It is useful to define the parameter $y = \sinh^2 t$. In terms of y, the wave-operator becomes

$$\Box = \frac{1}{|k|\alpha'}\left(4\partial_y y(1+y)\partial_y - \frac{1+y}{y}\partial_x^2\right)\,. \tag{3.6}$$

The equation of motion can then be written as the following eigenvalue equation

$$(\Box + m^2)\phi = 0\,. \tag{3.7}$$

Because $f(y) = e^{-ipx}y^{i|p|/2}\phi(x,y)$ satisfies a hypergeometric equation with parameters

$$a = j + 1 - \frac{i|p|}{2}\,, \qquad b = -j - \frac{i|p|}{2}\,, \qquad c = -i|p| + 1\,; \qquad z = -y\,, \tag{3.8}$$

we can easily find the solutions and expand the field in terms of them

$$\begin{aligned}\phi = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} d e^{ipx} y^{-i|p|/2} F\left(j + 1 - \frac{i|p|}{2}, -j - \frac{i|p|}{2}, -i|p| + 1; -y\right) + \\ + d^* e^{-ipx} y^{i|p|/2} F\left(j + 1 + \frac{i|p|}{2}, -j + \frac{i|p|}{2}, i|p| + 1; -y\right).\end{aligned} \tag{3.9}$$

Here

$$\begin{aligned} m^2 &= -\frac{4p^2}{|k|\alpha'} - \frac{16j(j+1)}{(|k|\alpha')} \\ &= -\frac{4p^2}{|k|\alpha'} + \frac{4 + 16s^2}{|k|\alpha'} \end{aligned} \tag{3.10}$$

can be interpreted as a mass squared. We assume that we are dealing with continuous representations of $\mathrm{SL}(2,\mathbb{R})$, or in other words, with delta-function normalizable wave-functions, for which $j = -\frac{1}{2} + is$ where $s \geq 0$.

Having solved the equation of motion, we turn to defining quantum fields at early and late times and obtain the corresponding vacua. This requires to identify positive and negative frequency modes at early and late times.

Early modes. In the early flat region (in Milne-like coordinates) near $t = 0$ (and t positive), the modes defined by the decomposition in equation (3.9) are approximately given by

$$\begin{aligned}\phi_p = d e^{ipx} t^{-i|p|} + \\ + d^* e^{-ipx} t^{i|p|}\,.\end{aligned} \tag{3.11}$$

This leads us to identify the normalized, positive frequency, early modes

$$u_p^e = \frac{1}{\sqrt{2|p|}} e^{ipx} y^{-i|p|/2} F\left(j + 1 - \frac{i|p|}{2}, -j - \frac{i|p|}{2}, -i|p| + 1; -y\right)$$

satisfying $(u_p^e, u_{p'}^e) = 2\pi\delta(p - p')$. (The index 'e' refers to early modes, while the index 'l' will refer to late modes.) We quantize the field in terms of these early modes by defining annihilation and creation operators as follows

$$\phi^e = \int \frac{dp}{2\pi}\left(a_p^e u_p^e + a_p^{e\,\dagger} u_p^{e\,*}\right). \tag{3.12}$$

The operators satisfy the usual commutation relations

$$[a_p^e, a_{p'}^{e\,\dagger}] = 2\pi\delta(p - p')\,. \tag{3.13}$$

The early vacuum $|e\rangle$ is annihilated by all a_p^e, $a_p^e|e\rangle = 0$.

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A few comments are in order.

- As we already remarked, the metric near $t = 0$ is given approximately by the flat metric in Milne-like coordinates. The early modes are positive and negative frequency with respect to the conformal time η, defined by $|t| = e^{\eta}$ near $t = 0$. Thus the early vacuum is approximately given by the conformal Milne vacuum (see [13]) in the region near $t = 0$.

- The corresponding frequencies near $t = 0$ are entirely determined by the momentum independently of the mass of the field. This fact can be explained as follows. The modes correspond to particles whose coordinate energy (conjugate to the time t) scales like $E_e^2 - \frac{p^2}{t^2} = m^2$ at early times. Thus as $t \to 0$, for fixed mass and momentum, the energy is dominated by the momentum dependent piece, $E_e \sim |p|/t$, which explains the result.

- Upon the analytic continuation $|p| \to i|n|$, and a change in the sign of the level, the positive frequency mode continues to the normalizable mode of the euclidean black hole (see e.g. [10]). Thus correlators in the early vacuum are expected to be related to the corresponding euclidean CFT correlators upon a suitable continuation.

Late modes. We define the late-vacuum at time $t = +\infty$ as follows. To obtain positive and negative frequency modes at late times, it is more convenient to write the solutions in the form

$$\begin{aligned}\phi = \int \frac{dp}{2\pi} b e^{ipx} y^{-j-1} F\left(j+1+\frac{i|p|}{2}, j+1-\frac{i|p|}{2}, 2j+2; -y^{-1}\right) + \\ + b^* e^{-ipx} y^j F\left(-j-\frac{i|p|}{2}, -j+\frac{i|p|}{2}, -2j; -y^{-1}\right).\end{aligned} \tag{3.14}$$

As $t \to +\infty$, the wavefunctions are given by

$$\phi = \int \frac{dp}{2\pi} b e^{ipx} (4)^{1/2+is} e^{-t} e^{-i2st} + b^* e^{-ipx} (4)^{1/2-is} e^{-t} e^{+i2st}, \tag{3.15}$$

which leads to the identification of the normalized, positive frequency, late times mode:

$$u_p^l = \frac{1}{\sqrt{4s}} e^{ipx} y^{-j-1} F\left(j+1+\frac{i|p|}{2}, j+1-\frac{i|p|}{2}, 2j+2; -y^{-1}\right). \tag{3.16}$$

This choice of modes defines the late vacuum following the steps described above. We make the following observations:

- In the (x,t) coordinates and at late times, the metric asymptotes to the flat Minkowski metric but the dilaton is linear in time. The string coupling constant is exponentially suppressed in time. The behavior of the late times modes as $t \to \infty$ matches onto the mode solutions for a timelike linear dilaton background [9].

- From the behavior of the modes as $t \to \infty$, we see that the parameter s defined by equation (3.10) determines the frequency of the modes or the energy of the corresponding particles at late times, $\omega = 2s$.

Particle creation. The early and late times modes can be related via a transformation in the arguments of the corresponding hypergeometric functions (notations and conventions are summarized in the appendix):

$$\begin{aligned}
u_p^l &= \frac{1}{\sqrt{4s}} e^{ipx} y^{-j-1} F\left(j+1+\frac{i|p|}{2}, j+1-\frac{i|p|}{2}, 2j+2; -y^{-1}\right) \\
&= \frac{1}{\sqrt{4s}} e^{ipx} \Bigg[y^{i|p|/2} \Gamma\left(\frac{-i|p|, 2j+2}{1+j-i|p|/2, j+1-i|p|/2}\right) \times \\
&\qquad \times F\left(j+1+\frac{i|p|}{2}, -j+\frac{i|p|}{2}, +i|p|+1; -y\right) - \\
&\qquad - y^{-i|p|/2} \Gamma\left(\frac{i|p|+1, -i|p|, 2j+2}{1-i|p|, 1+j+i|p|/2, 1+j+i|p|/2}\right) \times \\
&\qquad \times F\left(j+1-\frac{i|p|}{2}, -j-\frac{i|p|}{2}, -i|p|+1; -y\right) \Bigg]
\end{aligned} \tag{3.17}$$

leading to the identification of the Bogoljubov coefficients

$$\begin{aligned}
\alpha_p{}^{p'} &= \delta(p-p')\alpha(p) \\
\beta_p{}^{p'} &= \delta(p+p')\beta(p)
\end{aligned} \tag{3.18}$$

with

$$\begin{aligned}
\alpha(p) &= -\sqrt{\frac{|p|}{2s}} \Gamma\left(\frac{i|p|+1, -i|p|, 2j+2}{1-i|p|, 1+j+i|p|/2, 1+j+i|p|/2}\right) \\
\beta(p) &= \sqrt{\frac{|p|}{2s}} \Gamma\left(\frac{-i|p|, 2j+2}{1+j-i|p|/2, j+1-i|p|/2}\right).
\end{aligned} \tag{3.19}$$

The Bogoljubov transformation can be written as follows

$$\begin{aligned}
u^l(p) &= \alpha(p) u^e(p) + \beta(p) u^e(-p)^* \\
u^e(p) &= \alpha(p)^* u^l(p) - \beta(p) u^l(-p)^*
\end{aligned} \tag{3.20}$$

leading to the following relation in terms of the annihilation and creation operators

$$a_p^l = \alpha(p)^* a_p^e - \beta(p)^* (a_{-p}^e)^\dagger . \tag{3.21}$$

As usual the unitarity relation $|\alpha|^2 - |\beta|^2 = 1$ is satisfied. Since the early vacuum is not annihilated by a_p^l, it can be thought of as an excited state of the late vacuum, and therefore an observer at infinity concludes that particles have been created.

The total number of particles (for a given single field) produced from evolving the vacuum at early times to late times is given by

$$\langle e|N^l|e\rangle = 2\pi\delta(0) \int \frac{dp}{2\pi} |\beta(p)|^2 . \tag{3.22}$$

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The infinite factor $2\pi\delta(0)$ can be thought of as a volume factor and arises because of translational invariance in the spatial x-direction. Therefore, $\int dp|\beta(p)|^2$ determines the number of particles produced per unit volume. In our case, we obtain

$$|\beta|^2 = \frac{\cosh^2[\pi(|p|/2 - s)]}{\sinh(2\pi s)\sinh(\pi|p|)}\,. \tag{3.23}$$

Two limits of phase space parameters are interesting:

- In the limit of large momentum (and fixed mass), using the on-shell condition equation (3.10), the energy is given approximately by $\omega = 2s \sim |p|$ and so for large momentum $|\beta(p)|^2 \sim e^{-2\pi|p|} = e^{-2\pi\omega}$ leading to an exponential suppression in the production of high momentum particle pairs.

- In the small momentum limit, the energy is given approximately by the mass $\omega = 2s \sim m_{\rm eff} = (|k|\alpha' m^2/4 - 1)^{1/2}$ and so $|\beta(p)|^2 \sim \frac{\coth(\pi m_{\rm eff}/2)}{2\pi|p|}$. This leads to a divergence in the integral arising from the infrared region. For a massive field, the divergence is logarithmic. The surprising result is that the infrared divergence persists independently of the mass of the field.

Thus the particle spectrum produced is dominated by low momentum quanta. We can understand the result better by computing the rate of pair creation for a given mode. S-matrix elements between the early times vacuum and late times particle states are characterized by the quantity γ (or its complex conjugate) defined by [13]

$$\gamma^*(\omega) = -\frac{\beta(p)}{\alpha(p)^*} = \frac{\Gamma(2j+1)\Gamma^2(-j-i|p|/2)}{\Gamma(-2j-1)\Gamma^2(1+j-i|p|/2)}\,. \tag{3.24}$$

Thus the absolute value of $\gamma(\omega)$ characterizes the rate of creation of a given particle pair.[2] This is given by

$$|\gamma(\omega)|^2 = \frac{\cosh^2[\pi(|p|/2 - s)]}{\cosh^2[\pi(|p|/2 + s)]}\,. \tag{3.25}$$

For large momentum, $\omega = 2s \sim |p|$ and the rate is suppressed $|\gamma|^2 \sim e^{-2\pi\omega}$. For small momentum though, the rate is of order one: for $p \sim 0$, $|\gamma|^2 \sim 1$. Thus small momentum particles are readily produced.

It can be easily seen that when we analytically continue $|p| \to i|n|$, the amplitude in equation (3.24) becomes identical to the mini-superspace result for the reflection amplitude $\mathcal{R}(j,n)$ of a closed string mode (with quantum numbers (j,n)) in the euclidean black hole geometry [10]. The mini-superspace result is equivalent to the $k \to \infty$ limit of the relevant euclidean CFT two-point function. Thus at finite level k, we expect the exact answer for the rate amplitude to be obtainable from the euclidean CFT reflection amplitude upon the same analytic continuation accompanied also by a change in the sign of the level. Similar

[2] Conservation of x-momentum requires that pairs of particles of equal and opposite momentum are produced.

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results have been conjectured to hold in the cases of time-like bulk and boundary Liouville theories [1, 2]. Assuming that this is true, the exact amplitude is given by equation (28) multiplied by a pure phase factor given by [10]

$$ {\nu_b}^{i\omega} \frac{\Gamma(1+ib^2\omega)}{\Gamma(1-ib^2\omega)} \tag{3.26} $$

with $\nu_b = \Gamma(1-b^2)/\Gamma(1+b^2)$ and $b^2 = -1/(|k|+2)$. This implies that the rate (3.25) determined by the modulus of γ is exact even at finite level k. Thus the analysis of the particle spectrum we obtained can be carried through beyond the mini-superspace analysis at finite level k.

The results we found above indicate clearly that cosmological particle production leads to significant backreaction with a breakdown of perturbation theory. From the two dimensional field theoretic point of view, the problematic region is the infrared small momentum region. The divergence we found in the produced particle density can be associated to an instability of the early cosmological vacuum. For a massive scalar field, the divergence is milder and since low momentum particles are produced at a rate of order one in string units,[3] the space quickly fills with such non-relativistic particles. In other words, the space rapidly fills with dust-like matter, and presumably it eventually collapses. From the string theoretic point of view, the situation is more complicated and the resulting backreaction even more severe. In this case, we need to take into account the dependence of the effective mass on the momenta along the extra spatial directions and the exponentially large Hagedorn density of string states at large energy (and oscillator level). We discuss this case next.

Embedding in string theory. We saw that for a fixed (cosmological) mass of a given closed string field, and at large energy, the rate of particle production is either exponentially suppressed as a function of the energy $|\gamma|^2 \sim e^{-2\pi\omega}$ or of order one $|\gamma|^2 \sim 1$, depending on whether the momentum along the x-direction is large or small. When we further add appropriate factors to build a critical string theory, e.g. by adding a euclidean black hole conformal field theory (at level $|k|+4$) and a twenty-two dimensional flat space factor, the rate of particle production at large energy will be enhanced due to the exponentially large Hagedorn density of closed string states: $\rho(\omega) = e^{\frac{\omega}{T_H}}$. In this example, $T_H = \frac{1}{2\pi}\sqrt{\frac{6}{D-2}} = \frac{1}{4\pi}$, and so the total rate of particle production $\rho(\omega)|\gamma|^2$ at large energy is exponentially divergent.[4]

In fact it is easy to identify regions in phase space parameters where the particle production rate at large energy is exponentially divergent, irrespectively of the details of the string theory model we construct. One such region is the region where the oscillator level N, and hence the energy, of closed string states that are produced are very large

[3]The time-scale for the breakdown of perturbation theory is set by the string scale. No matter how small the initial value of the string coupling is, once enough particles are produced, they will lead to significant backreaction.

[4]We do not expect the details of the spins and polarizations of the closed string fields along the extra directions to alter this conclusion.

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while all momenta are kept fixed (and small). The rate of particle creation (per field) is of order one in this regime, while the degeneracy of states (as a function of the square root of the oscillator level) grows exponentially with an exponent which is (twice) the inverse of the closed string Hagedorn temperature (see e.g. [15]). The type of matter produced in this regime consists of heavy, almost non-relativistic strings and is similar to the "tachyon matter" produced during the decay of unstable branes [20]–[22]. Within a time-scale of order the string length, enough particles are produced to lead to large backreaction and a breakdown of perturbation theory. Understanding the fate of the cosmology then requires understanding strings at very high densities.

The breakdown of perturbation theory indicates that non-perturbative effects are important in understanding the fate of these cosmological models. One expectation is that such non-perturbative effects will cut-off the exponentially growing density of string states. Given this, particle production in string cosmology can be turned into an advantage in constructing realistic models of inflationary cosmology since such particle production may result in re-heating the universe [23, 24] after an early inflationary phase.

It is useful (and sometimes appropriate) to think of the inverse of the exponent in the rate amplitude as a cosmological temperature. In the example we studied, the cosmological temperature is momentum dependent, and for small momenta, it is clearly far above the Hagedorn temperature since then the cosmological temperature is infinite. Another recent example where a similar phenomenon was observed is the $0 < k < 2$ cosmology of [16]. In time-like bulk Liouville theory, the situation is slightly better in that the cosmological temperature is only double the Hagedorn temperature (still leading to large backreaction) [2]. This is the regime in our case as well for high x-momentum.

String theory seems to naturally produce cosmological solutions in which the cosmological temperature is larger than the Hagedorn temperature, leading to large backreaction.[5] One can ask whether there are ways to lower the cosmological temperature, or to heighten the Hagedorn temperature. We note that the cosmological temperature depends on the choice of initial state and initial surface. The leading (mini-superspace) behavior may also receive higher curvature corrections which at strong curvature could significantly alter the cosmological production rate.[6] Heightening the Hagedorn temperature can be achieved in non-critical strings [17] (although the linear dilaton behavior would need to be short-cut by a Liouville-like potential to avoid strong coupling problems). It would be interesting to understand better the generic constraints on string theory that lead to these UV catastrophes in cosmology.

Note that our vacuum also generates a lot of particles in the IR regime (at small x-momentum p). This indicates that we would need to modify the definition of the early cosmological state in the infrared, for a proper time evolution. Indeed, as we argued the onset of the instability occurs at an early timescale. It would be interesting to understand the physical consequences of such an IR regularization at early times.

[5]We thank Dan Israel for discussions on this topic.

[6]However, as we indicated above, by using the connection between the amplitude for pair creation and the exact euclidean reflection amplitude, one can show that no such correction occurs for the particular cosmology that we studied.

Finally, we add a word on methodology. Implicitly, we have taken the attitude that a second quantized string theory in this cosmological background can be defined, such that we can study a collection of second quantized fields with standard methods developed for quantum fields in curved space. This should be contrasted with computations in the first quantized approach, in which it would only seem to make sense to ask questions about the stringy S-matrix. It would be interesting to investigate the formulation of string field theory on time-dependent backgrounds, and to see how these two attitudes should be reconciled.

After these remarks on aspects of bulk string cosmology, we turn to the study of cosmological effects due to non-perturbative time-dependent D-branes.

4. Particle production in the presence of a source

In this section, we develop a formalism to take into account simultaneous particle creation due to cosmological evolution on the one hand and due to the presence of a time dependent source (in a linearized approximation) on the other hand. To derive the relevant particle production amplitude, we follow e.g. [18].

The most general solution to the wave equation in the presence of the source can be written as follows

$$\phi(x,t) = \phi_e(x,t) + i\int dt'dx'\sqrt{-g}e^{-2\Phi}G_R(x,t;x',t')\rho(x',t')\,, \tag{4.1}$$

where $\phi_e(x,t)$ solves the homogeneous, sourceless wave equation and $G_R(x,t;x't')$ is the retarded propagator satisfying

$$(\Box + m^2)G_R(x,t;x',t') = -i\frac{\delta(x-x')\delta(t-t')}{\sqrt{-g}e^{-2\Phi}}\,. \tag{4.2}$$

The retarded propagator is given by

$$G_R(x,t;x',t') = \theta(t-t')\langle e|[\phi(x,t),\phi(x',t')]|e\rangle\,. \tag{4.3}$$

Here $|e\rangle$ refers to the early vacuum. Therefore, it can be formally computed in terms of the Wightman functions $G(x,t;x',t') = \langle e|\phi(x,t)\phi(x',t')|e\rangle$ and similarly for $G(x',t';x,t)$. In terms of the early modes the Wightman function is given explicitly by

$$G(x,t;x',t') = \int\frac{dp}{2\pi}\frac{1}{2|p|}e^{ip(x-x')}f^j_{|p|}(t)\left[f^j_{|p|}(t')\right]^*\,, \tag{4.4}$$

where we defined $f^j_{|p|}(t) = y^{-i|p|/2}F(j+1-i|p|/2,-j-i|p|/2,-i|p|+1;-y)$ with $y=\sinh^2 t$.

Let us suppose that the source starts and stops. Then if we wait until all of the source is in the past, the field solution can be written as follows

$$\phi(x) = \int\frac{dp}{2\pi}\left[a^e_p + \frac{i}{\sqrt{2|p|}}\tilde{\rho}(j(p),p)\right]u^e_p + \text{h.c.}, \tag{4.5}$$

where $\tilde{\rho}(j,p)$ is the Fourier transform of the source defined by[7]

$$\tilde{\rho}(j,p) = \int dtdx\sqrt{-g}e^{-2\Phi}\rho(t,x)e^{-ipx}(f^j_{|p|}(t))^* \qquad (4.6)$$

and evaluated at energies and momenta satisfying the on shell condition, eq. (3.10). Only the on-shell part of the source contributes to the late quantum field. In the case when a localized source persists at very late times, the above formula applies only in the limit $t \to \infty$, away from the source.

Using the Bogoljubov coefficients, we can write the early modes in terms of the late field modes. Since $u^e_p = \alpha(p)^* u^l_p - \beta(-p)(u^l_{-p})^*$, we obtain after substitution

$$\phi(x,t) = \int \frac{dp}{2\pi}\left((a^e_p + \frac{i}{\sqrt{2|p|}}\tilde{\rho}(p))(\alpha(p)^* u^l_p - \beta(-p){u^l_{-p}}^*) + \text{h.c.}\right). \qquad (4.7)$$

We can now identify the coefficient of u^l_p as the late time annihilation operator including the action of the source to obtain

$$a^{l,\rho}_p = (a^e_p + \frac{i}{\sqrt{2|p|}}\tilde{\rho}(p))\alpha(p)^* - ({a^e_{-p}}^\dagger - \frac{i}{\sqrt{2|p|}}\tilde{\rho}(-p)^*)\beta(-p)^*. \qquad (4.8)$$

We see that we recuperate standard results for cosmological particle production when the source is zero [13], and when the early and late modes in the cosmology are identical, we recuperate standard production of particles by a classical source [18]. Finally, we compute the total number of particles produced:

$$\langle e|N^l|e\rangle = \int \frac{dp}{2\pi}\left(\mathcal{V}|\beta(p)|^2 + \frac{1}{2|p|}|\tilde{\rho}(p)\alpha^*(p) + \tilde{\rho}^*(-p)\beta(-p)^*|^2\right). \qquad (4.9)$$

5. The brane source

We wish to study D0-branes in the cosmology in the mini-superspace approximation. The effective metric for such a brane is given by

$$\frac{ds^2}{g_s^2} = dudv\,, \qquad (5.1)$$

and so it is flat. Because the effective metric is flat, a D0-brane follows a straight line trajectory in the (u,v) plane. One example is the brane at fixed $X^1 = (u+v)/2 = c$. All other brane trajectories can be obtained by performing a boost on this brane

$$e^{-x_0}u + e^{x_0}v = 2c\,. \qquad (5.2)$$

From the point of view of the string frame metric, the branes follow accelerated (non-geodesic) trajectories. The reason is that they experience a force from the changing dilaton background. In terms of the (x,t) coordinates, the 2-parameter family of the brane

[7] We summarize completeness/orthogonality relations in appendix A.

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trajectories can be written as follows

$$\sinh(x - x_0) \sinh t = c \,. \tag{5.3}$$

Since such probe branes follow straight lines in the (u, v) plane, they seem to be oblivious to (naive) space-time singularities.[8]

The description of such a brane from the point of view of the cosmological observer using the (x, t) coordinates is as follows. For $c > 0$, the brane enters the cosmological region crossing the horizon $u = 0$ at finite null time $v_0 = 2ce^{-x_0}$. In terms of the (x, t) coordinates, this event occurs at $t = 0$, $x \to \infty$. The initial x-velocity of the brane is infinite[9] (and negative). Subsequently the brane decelerates until it localizes at $x = x_0$ as $t \to \infty$. A similar description can be obtained when $c < 0$. The fact that the brane decelerates from the point of view of this observer has a natural explanation. The dilaton background field is time dependent such that it decreases as t goes from zero to infinity. During late times, the dilaton field decreases exponentially and the metric is flat – in the (x, t) coordinates the background asymptotically approaches the linear dilaton background as $t \to \infty$. As a result, the brane gets heavier and heavier during cosmological evolution. Conservation of momentum then implies that it must decelerate.

Such a brane leads to a delta-function source for closed string fields localized on its trajectory. The source is time dependent and part of it will be on-shell. So we naturally expect on-shell emission of closed strings.[10] To obtain the precise form of the source, we examine the coupling of the brane to the dilaton fluctuations $\delta\Phi$ in the linearized approximation. This is given by

$$\int dt dx \sqrt{-g} e^{-2\Phi} \rho(x, t) (\delta\Phi) \,, \tag{5.4}$$

where

$$\rho(x, t) = \frac{\delta(x - \bar{x}(t))}{l_s (c^2 + \sinh^2 t)^{1/2}} \tag{5.5}$$

and $\bar{x}(t)$ denotes the probe trajectory. Similar couplings are assumed for other closed string fields as well. To get the amplitude relevant for particle production, we need to Fourier transform the source with respect to the early, or late time modes and evaluate it on shell, as described in section 4.

We proceed to analyze the behavior of the Fourier transform of the source both in terms of the early and the late times modes. We have been able to compute the source in momentum space for the case $c = 0$. This family of branes is special in that the branes remain localized in the (x, t) coordinates and the time dependence results solely from the changing mass of the branes. Fourier transforming with respect to the early modes, we

[8] This behavior is directly related to the fact that D0-branes in the two-dimensional black hole pass through singularities without much ado [19].

[9] The proper velocity of the brane is finite as can be seen from the behavior of the metric near $t = 0$.

[10] We note that both the annulus amplitude and the one point function of a closed string vertex operator on the disk are not suppressed by the diminishing string coupling since they are of order g_s^0.

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obtain

$$\tilde{\rho}^*(j,p) = \int_{-\infty}^{+\infty} dx \int_0^\infty dt \cosh t \sinh t \delta(\sinh(x-x_0)\sinh t) e^{ipx}$$
$$(\sinh t)^{-i|p|} F\left(j+1-\frac{i|p|}{2}, -j-\frac{i|p|}{2}, -i|p|+1; -\sinh^2 t\right) =$$
$$= \frac{1}{2} e^{ipx_0} \Gamma\left(\frac{-is, is, -i|p|+1}{is - i|p|/2 + 1/2, -is - i|p|/2 + 1/2}\right), \tag{5.6}$$

while the Fourier transform with respect to the late modes yields

$$\tilde{\rho_l}^*(j,p) = \int_{-\infty}^{+\infty} dx \int_0^\infty dt \cosh t \sinh t \delta(\sinh(x-x_0)\sinh t) e^{ipx}$$
$$(\sinh t)^{-2j-2} F\left(j+1+\frac{i|p|}{2}, j+1-\frac{i|p|}{2}, 2j+2; -\sinh^{-2} t\right) =$$
$$= \frac{1}{2} e^{ipx_0} \Gamma\left(\frac{is, 1+2is, 1/2+i|p|/2, 1/2-i|p|/2}{is + i|p|/2 + 1/2, is - i|p|/2 + 1/2, 1+is}\right). \tag{5.7}$$

The phase factor e^{ipx_0} in the amplitudes (5.6) and (5.7) can be simply understood as a consequence of translational invariance along the x-direction. The localized brane breaks this symmetry spontaneously. For a discussion of the Fourier transform of the generic source (and the comparison to the exact one-point function for D1-branes in the euclidean cigar geometry), we refer to appendix B.

We can now turn to the computation of the number of on-shell particles produced by the D0-brane. The generic result in equation (4.9) contains the bulk term, which we have discussed in detail in a previous section, and the source term. The latter is relatively suppressed by an extra factor proportional to the inverse volume of the spatial x-direction. This is because the source is localized in this direction (and, in contrast to the bulk particle production, the source production does not occur homogeneously in space). For the number of particles produced by the source, we obtain

$$\int \frac{dp}{2\pi} \frac{1}{2|p|} |\tilde{\rho}(p)\alpha^*(p) + \tilde{\rho}^*(-p)\beta(-p)^*|^2 =$$
$$= \int \frac{dp}{2\pi} \left[\frac{\pi}{8s^2} \frac{\cosh[\pi(s+|p|/2)]\cosh[\pi(s-|p|/2)]}{\sinh(2\pi s)\cosh^2(\pi|p|/2)}\right]. \tag{5.8}$$

The energies and momenta satisfy the on-shell condition equation (3.10). The computation can either be done by using the Fourier transform with respect to the early modes, and then using the formalism of section 4, or equivalently, by using directly the Fourier transform with respect to the late modes (which automatically takes into account the cosmological evolution encoded in the Bogoljubov coefficients).

The following comments are in order:

- In the limit of small x-momentum, the energy is essentially given by the effective cosmological mass, and the amplitude squared becomes $|\rho_l|^2/4s \sim \frac{\coth(m_{\text{eff}}/2)}{m_{\text{eff}}^2}$. Thus for massive particles the production amplitude due to the brane source in this regime is of order one, while for massless particles we get an infrared divergence. The latter can be associated with the static long range fields produced by the source. The effective cosmological mass depends both on the momenta along the extra directions and the oscillator level of the emitted string state.

- In the limit of large x-momentum and fixed effective cosmological mass, the on shell condition is given by $\omega = 2s \sim |p|$. In this limit we obtain an extra exponential suppression factor $|\rho_l|^2/4s \sim \omega^{-2}e^{-\pi\omega}$ in the amplitude of particle production. Notice that the exponent of the suppression factor is only half of that corresponding to bulk pair production in the large x-momentum regime.

- When we embed in string theory, we need to take into account the enhancement factor due to the exponentially large density of states at large energy. Now in the case of the brane source, the closed string states that are produced are left/right symmetric and their density is controlled by the open string density [21].

The considerations above indicate that the spectrum of closed strings emitted by the brane consists predominantely of small momentum highly massive states. If the brane is wrapped along extra flat directions, conservation of momentum along these directions forbids emission of strings with non-zero momentum along these extra directions. And for localized branes along flat extra directions, the amplitudes (for scalars) are modified by phase factors such as e^{iky_0}, with the dependence of the production amplitude squared on the momentum along these directions arising through the effective cosmological mass. As we saw, for higher effective mass there is a power law suppression factor. Since then there is an exponentially growing density of closed string states as a function of the oscillator level, the spectrum consists mostly of slowly moving, highly massive particles. Hence the emitted matter remains localized close to the brane. Presumably the mass gained by the brane, after backreaction is included, results in the production of such massive strings. For massive string states, it is important to consider finite k corrections to the mini-superspace analysis we described. We comment on how to embed in string theory further below.

As can be seen from the analysis in appendix B (see equation (B.1)), the small x-momentum regime of the $c \neq 0$ branes is better in that an exponential suppression factor appears for high cosmological mass (and hence energy) given by $e^{-\pi\omega(1-2q/\pi)}$, where the parameter q is defined by $\cos q = |c|$. For $c = 0$, $q = \pi/2$, and this factor is one, while for $|c| < 1$, $2q/\pi < 1$, and an exponential suppression factor survives. The reason for this phenomenon can be understood intuitively as follows. When $c \neq 0$, these branes enter the cosmological region with initial x-momentum, and the resulting radiation of strings with zero x-momentum should now be less favored relatively to the emission of non-zero momentum strings, always in comparison for the relative strengths in the $c = 0$ case. However, the suppression factor is not small enough to compete with the exponentially large Hagedorn density of closed string states in most string theory models one can construct. For $|c| > 1$, the parameter q is imaginary and appears through an oscillatory factor in the amplitude squared. For these cases the suppression factor seems to be the biggest possible. We note that the trajectories of these branes, when continued to the other regions of the cosmology, cross the timelike naked singularities.

Embedding in string theory. Again, we can embed the model in string theory by adding a euclidean black hole factor and extra flat space factors to the target space. It is then straightforward to identify a region of phase space in which the energy grows like the oscillator level (with all momenta fixed (and small)), where the particle production amplitude is not suppressed significantly, and where the exponential degeneracy of states leads to an exponentially divergent number of particles produced. As we pointed out above, the exponential growth of the relevant states is governed by the open string density. (One should compare this to the less catastrophic behavior observed in [21] for the case of unstable branes.)

When we perform the exact analysis beyond mini-superspace, we observe that an extra factor (see appendix B and [10])

$$\Gamma\left(1+\frac{i\omega}{k-2}\right) \tag{5.9}$$

appears in the amplitude for one-closed string production, which leads to an extra suppression factor $e^{-\frac{\pi\omega}{|k|+2}}$ in the particle production rate for k negative. Sadly, this will be insufficient to lead to finite particle production rates. There is a narrow window of opportunity, at very small k (close to 2), i.e. a highly curved cosmology of the type discussed in [16], in which one could swamp the Hagedorn density with this suppression factor. It is interesting to note that no such opportunity appeared in the closed string sector.

Notice again the similarity of the resulting particle spectrum to the "tachyon matter" produced during the decay of unstable D-branes in flat space [20]. It would be interesting to see if we can identify a tachyonic mode in the effective field theory on the brane (compare e.g. to [19]) and see if we can map the problem to a tachyon condensation problem on the brane as in [5].

6. Conclusions

We have argued for an analytic continuation of the euclidean $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ conformal field theory and its interpretation as the future part of an interesting two-dimensional cosmology. We thus manage to incorporate all α' corrections to the relevant amplitudes discussed. We have identified and discussed a generic bulk backreaction problem due to a cosmological temperature higher than the Hagedorn temperature. The introduction of a non-perturbative source, such as a D0-brane, leads to additional particle production, which might be kept under control in highly curved backgrounds. The formalism we developed for the interplay between a cosmology and a time-dependent source should be useful for other stringy cosmological models as well.

We have touched upon many directions of future research in the bulk of our paper. Other regions of the lorentzian cosmology (and D-branes propagating in these regions) can be continued to other euclidean conformal field theories, which in turn await an interpretation in the lorentzian realm (e.g. the D1-branes in the minimal models). The patching

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together of different "euclidean conformal field theory regions" after suitable analytic continuation, and an understanding of the matching of D-brane trajectories on the boundaries of these regions is an interesting problem. An easier first step might be found in the analysis of the boundary conditions induced at the timelike singularities by particular euclidean conformal field theories.

More generically, one can investigate the feasibility of formulating string field theory in cosmological backgrounds, the construction of models with cosmological temperature below the Hagedorn temperature, and the relationship between open and closed string vacua induced by non-trivial consistency relationships between bulk and boundary euclidean conformal field theories.

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A. Formalities

Formulas. We introduce the following notation for a product and quotient of Gamma-functions

$$\Gamma\left(\frac{t_1, t_2, \ldots, t_n}{b_1, b_2, \ldots, b_m}\right) = \frac{\Gamma(t_1)\Gamma(t_2)\ldots\Gamma(t_n)}{\Gamma(b_1)\Gamma(b_2)\ldots\Gamma(b_m)}\,. \tag{A.1}$$

We use the transformation rule (i.e. rule for analytic continuation) for hypergeometric functions

$$\begin{aligned}(-z)^{-a}F(a, a+1-c, a+1-b; z^{-1}) = {}& \Gamma\left(\frac{1-c, a+1-b}{1-b, a+1-c}\right) F(a,b,c;z) - \\ & - \Gamma\left(\frac{c, 1-c, a+1-b}{2-c, c-b, a}\right) e^{i\pi(c-1)} z^{1-c} \times \\ & \times F\left(a+1-c, b+1-c, 2-c; z\right), \end{aligned} \tag{A.2}$$

and we make good use of the integral formulas [26]

$$\int_0^\infty F(a,b,c;-z) z^{-t-1} dz = \Gamma(\frac{a+t, b+t, c, -t}{a, b, c+t}) \tag{A.3}$$

which is valid when $Re(t) < 0$, $Re(a+t) > 0$, $Re(b+t) > 0$ and c should not be a negative integer, and

$$\begin{aligned}\int_0^\infty x^{c-1}(x+z)^{-\sigma} F(a,b,c;-x) dx = {}& \Gamma\left(\frac{c, a-c+\sigma, b-c+\sigma}{\sigma, a+b-c+\sigma}\right) \times \\ & \times F(a-c+\sigma, b-c+\sigma, a+b-c+\sigma; 1-z)\end{aligned} \tag{A.4}$$

which holds when $\mathrm{Re}(c) > 0$, $\mathrm{Re}(a-c+\sigma) > 0$, $\mathrm{Re}(b-c+\sigma) > 0$ and $|\arg(z)| < \pi$.

Completeness. We assume the following completeness/orthogonality relations for the early modes (which should be compared to analogous relations for their euclidean counterparts [25]):

$$\begin{aligned}
\rho(y,x) &= \int \frac{dp}{2\pi} \int_{-\frac{1}{2}+iR^+} \frac{dj}{2\pi} \frac{1}{|N(j,p)|^2} \left[\tilde{\rho}(j,p) e^{ipx} f^j_{|p|}(y) + \text{c.c.}\right] \\
\tilde{\rho}(j,p) &= \int \frac{dydx}{2} \rho(y,x) e^{-ipx} (f^j_{|p|}(y))^* \\
\int \frac{dydx}{2} &(e^{ipx} f^j_{|p|}(y))(e^{-ip'x}(f^{j'}_{|p'|}(y))^*) = |N(j,p)|^2 (2\pi)^2 \delta(j-j')\delta(p-p') \,, \qquad \text{(A.5)}
\end{aligned}$$

where we recall that $y = \sinh^2 t$. The factor $|N(j,p)|$ is a normalization factor and the measure $\sqrt{-g}e^{-2\Phi}dt = dy/2$.

B. The generic source in momentum space

In this appendix, we make some observations that lead to a guess for an expression for the source in momentum space for a brane with a generic trajectory. We will base our guess on the expression for the one-point function of the D1-brane on the cigar geometry, which is the natural euclidean analog of our D0-brane in the cosmology, as can be seen by comparing their trajectories.

Before turning to this problem, we note an additional partial result. We can determine the Fourier transform of the source at zero momentum ($p = 0$) for a generic trajectory (parametrized by c) as follows

$$\begin{aligned}
&\int_{-\infty}^{+\infty} dx \int_0^\infty dt \cosh t \sinh t \delta(\sinh(x-x_0)\sinh t - c) e^{ipx} \times \\
&\times (\sinh t)^{-i|p|} F\left(j+1-\frac{i|p|}{2}, -j-\frac{i|p|}{2}, -i|p|+1; -\sinh^2 t\right) = \\
&=_{|p=0} \frac{1}{2s} \frac{\cos(2j+1)q}{\sinh \pi s} \,, \qquad \text{(B.1)}
\end{aligned}$$

where we defined $\cos q = |c|$ for $|c| < 1$ and $q \in [0, \pi/2]$.

With this extra trump in our hand, we turn to the full problem. We note that the semi-classical one-point function for the D1-brane in the cigar [10] can be rewritten, after multiplying the one-point function by the inverse of the normalization factor for the euclidean eigenfunctions, as

$$\begin{aligned}
\langle \chi_{\text{closed}} \rangle &= e^{in\theta_0} \frac{\sin \pi(\frac{1}{2} - is + |n|/2)}{2s \sinh 2\pi s} \Gamma\left(\frac{|n|+1}{1/2+is+|n|/2, 1/2-is+|n|/2}\right) \times \\
&\times \left(e^{-r(2j+1)} + e^{i\pi n} e^{r(2j+1)}\right), \qquad \text{(B.2)}
\end{aligned}$$

which at $r = 0$ can be rewritten, for n even, as

$$\langle \chi_{\text{closed}} \rangle = e^{in(\theta_0 + \frac{\pi}{2})} \frac{1}{2s \sinh \pi s} \Gamma\left(\frac{|n|+1}{\frac{1}{2}+is+|n|/2, \frac{1}{2}-is+|n|/2}\right). \qquad \text{(B.3)}$$

We expect the analytic continuation to map $|n|$ to $-i|p|$ (since this is the prescription that maps normalizable euclidean modes to early positive frequency modes), and indeed, we find a match with our results for the Fourier transform at $c = 0$ (up to an overall constant).

To address the generic case, it is important to notice that the D1-brane on the cigar intersects most circular cross sections of the cigar in precisely two points, while the D0-brane in the cosmology only intersects a spatial slice at one point. Naively, analytically continuing the cigar source to the lorentzian theory would lead to a doubling of D-branes.

To obtain the correct one-point function for a single source, we propose to perform the following rewriting of the one-point function

$$\begin{aligned}\langle \chi_{\text{closed}} \rangle &= e^{+i\pi n/2} e^{in\theta_0} \frac{1}{2s \sinh 2\pi s} \Gamma\left(\frac{|n|+1}{1/2+is+|n|/2, 1/2-is+|n|/2}\right) \times \\ &\quad \times \left(\cosh s(\pi + 2ri) e^{in\pi} + \cosh s(\pi - 2ri)\right). \end{aligned} \tag{B.4}$$

We wish to identify the two terms with two possible sources in the lorentzian theory, at c and $x_0 = i(\theta_0 + \pi)$ and at $-c$ and $x_0 = i\theta_0$ respectively. The results now match onto the results we obtained for generic $|c| = \cos q$ at $p = 0$, taking into account that $c = \sin r'$ and $q = \pi/2 - |r'|$, for $r' = ir \in [-\pi/2, \pi/2]$, and taking into account just the corresponding single source term.

Although we haven't been able to do the integral exactly, we thus propose an expression for the FT of the generic case (depending on the sign of r'):

$$e^{ipx_0} \frac{1}{4s \sinh \pi s \cosh \pi s} \Gamma\left(\frac{-i|p|+1}{1/2+is-i|p|/2, 1/2-is-i|p|/2}\right) \cosh \pi s \left(1 \mp r' \frac{2}{\pi}\right). \tag{B.5}$$

We estimate that the generic integral that should lead to this result can presumably be performed by delving more deeply into the representation theory of $\mathrm{SL}(2,\mathbb{R})$. We observe that the large energy behavior of the one-point function crucially depends on the value of the parameter r'.

This guess for the full semi-classical one-point function can now be extended to the exact one-point function for the D0-brane in the cosmology, by repeating the above manipulations on the known [10] exact result for the one-point function of the D1-brane in the euclidean cigar . This leads to the extra factor in the exact result which we quoted in the bulk of our letter.

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